Thinking Mathematically Podcast Transcripts

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All Kinds of Minds®

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Thinking Mathematically Podcast Transcript

Mathematics: A Unique Fusion of Moving Parts

Dependence on the synchronous use of multiple sub-skills and functions

Whenever we want to think about what's involved in children really trying to master the challenge of mathematics, we have to recognize that there are so many unique things about Math in the curriculum that really differ significantly from other skills and other subject areas that children contend with. I'd like to just mention some of those differences because they can have a striking impact on the way kids perform. First of all, Math has a lot of moving parts, and success in Math depends on being able to integrate those moving parts which really consist of sub-skills such as multiplication, division, addition, and subtraction; also some very key underlying brain functions like Memory and Language and Spatial abilities. And these various functions and sub-skills have to be coordinated. They have to sort of come together and they have to operate in synchrony. You can't have one that works much faster than the others. So, Math requires a lot of orchestration on the part of a child's brain as it tries to pull together these multiple critical sub-components.

Demand for convergent thinking

Another unique part of Math that its one of the most convergent subjects in the curriculum. By that I mean that most of the time in mathematics there is only one correct answer. In Social Studies and in English classes many different interpretations are acceptable. On a quiz there are many different answers that would be considered meritorious. But in Math, there is the right answer. And certain kinds of minds don't feel particularly comfortable with having to be that convergent. They'd much rather diverge and explore possible responses. So that can be a source of some anxiety as children are learning Math. Particularly certain kinds of wandering minds that may not be highly convergent in their style of thinking.

Use of verbal and non-verbal thinking; mental imagery as a back up to language

Another kind of blend that has to go on, another really critical part of doing well in mathematics is being able to alternate between spatial thinking and language. Being both verbal and non-verbal so while you are learning a new skill or trying to solve a problem, it's helpful to be able to verbalize what you are doing to kind of put the process into words but at the same time there needs to be some mental imagery - some kind of visual back up to your language. So that ability to have sight and sound to kind of have a soundtrack to your video while you're solving or thinking about math is crucial. And one other element finds its way into Math which is sequencing – the ability to really appreciate serial order so that you do mathematical steps in the right order. So that your sense of number is very much sequentially arranged and so we have to add to the mix Language and Spatial Thinking along with sequencing ability. And if one of those is relatively weak in a child, his or her mathematical mastery can be pretty shaky.

Manipulation of symbols

And here's another ingredient that plays a significant role in mathematical mastery – and that's the use of symbols. Certainly in elementary school most other subjects are really sparse in their use of symbols. But mathematics is full of things that stand for something else. It's a highly symbolic subject. And one has to be able to step back and really somehow overcome one's commitment to the concrete world and start to manipulate things that are symbolic. As kids get older it might be operational signs. As they enter toward Algebra, unknown x's and y's become sort of the raw materials of symbolism in mathematics.

Frequent movement between abstract and concrete thinking

And yet kids also have to think on an abstract basis and they have to go back and forth from abstract to concrete thinking. So a problem may be represented on a graph and at the same time it could be characterized within a word problem, and that ability to go back and forth between very concrete thinking such as one might be able to at least launch in a word problem. Jack has 4 apples and Mary has 6 apples. How many would they have if they shared them equally? That allows for some concrete thinking. Also, when you go to the store, calculating how much change you'll need to get when you buy something, that's very concrete and yet it can move beyond that and all of a sudden that concrete thinking has some very abstract manifestations as you think about the process of subtraction and what different coins represent.

Highly cumulative learning

Another unique characteristic of mathematics is its highly cumulative nature. In an English class you may not have to struggle very hard to remember in February what you learned in November because you are reading a different story in February and it's not particularly dependent on the story your read in November. In mathematics, everything is layered. What comes up in the Spring is very much based on how well you learned what you needed to learn in the Fall. And that cumulative learning can be very intimidating to certain students. It places heavy demands on memory function and also on the intensity and activity with which you learn the information [or] skill in the first place.

Integration of comprehension and application

Another thing that is worthy of mention is the very interesting kind of interaction that goes on in mathematics between comprehension and application. You know it's possible, very possible to make use of something in Math without really understanding it. It's possible to add fractions, subtract fractions, and divide fractions without having any grasp whatsoever on what a fraction is. In fact, one has to worry a bit when kids prematurely apply something – when they are starting to use something they don't understand. Because in the long run that can lead to some real tenuousity – to a shaky grasp – and it can all come crashing down as the demands intensify. So as we work with kids, it's so important not to leap into applications before we've established really good comprehension. And in fact throughout the mathematical career of a child there's this constant tension between knowing how to use something and knowing what you're doing.

Parallel requirements of Understanding and Recall

Last but not least, I think, and certainly related to application versus comprehension, is the issue of comprehension and memory as they interact in Math. It's kind of a double-helix throughout ones mathematical learning. It's important to really understand the processes, the concepts, the vocabulary, and the various issues that keep coming up in mathematical learning. But at the same time, there's an awful lot of memory load and keeping that balance between remembering and understanding is sometimes a very different equilibrium for kids to form and sustain; and yet the balance between the understanding and recall is at the core of mathematics. And the truth is you have to be good at both.

You know it's possible to look at math accomplishments almost like some kind of dish that a chef is preparing in a restaurant. In other words, math has ingredients. Ingredients in math are the different neurodevelopmental functions that have to converge and collaborate in order for certain mathematical skills to be acquired, and in order for really effective mathematical problem solving to take place. We can go back to the earlier distinction we made between understanding and recall and really divide the neurodevelopmental functions into those two realms. So let's begin with the kind of comprehension underpinnings of mathematics, and three of our neurodevelopmental concepts are especially germane when it comes to mathematical comprehension. Those three are **Language**, **Higher Order Cognition**, and **Spatial Ordering**. So let's take those one at a time …

Language

Sometimes it's easy to overlook the fact that Math is as much of a language as English class or Social Studies. There are a lot of verbal inputs and outputs in Mathematics. In some ways the whole subject of Math is a language. So let's take that apart a little bit, and say what are the language components of mathematics.

> ***Math-specific semantics (e.g., numerator, denominator)***
First of all, there's the semantics of math that can be very elusive for many people. A lot of the words in math are not self-evident, words like numerator, denominator, and hypotenuse. A lot of them don't actually make much sense. For example we have something called an equilateral triangle that has three equal sides. Then all of a sudden one pops up – an isosceles triangle – that has only two equal sides. A lot of kids would prefer to call it a bilateral triangle! But a lot of that language is not totally rational and you have to sort of integrate it even though in reality it does not make a lot of linguistic sense.

> ***Processing verbal explanations/ modeling as a form of math instruction***
Another part of the language demands of math is the ability to process all the verbal explanations while the teacher is explaining something, [while she] is really telling your head to do something. And by the way, there are some kids who would much rather prefer a demonstration model to a verbal explanation. Being able to process a verbal explanation requires good sentence comprehension. It requires kids to be able to process very quickly because verbal explanations come at a much more rapid rate than studying a correctly solved problem or using some other form of demonstration, so speed of processing becomes important with the verbal explanatory part of mathematics.

> ***Comprehension of word problems***
And then there's the real bugaboo called word problems. Word problems contain some of the most complex sentences a young child ever encounters, anywhere. For example, a 4-year old feels very comfortable with the notion of the order of mention strategy which stipulates that when your mom tells you to do something, she tells you to do it in the order in which she wants you to do it. That becomes very much part of a kid's thinking about sentence structure.

Word problems keep violating the order of mention strategy. They tell you what to do, but they don't tell you to do it in the order in which you're going to need to do it. That and a lot of other syntactical kinds of contortions can sometimes make mathematical learning very difficult, especially if you're not terribly adept with language at the sentence level.

> ***Symbolic language***
> Another part of mathematical linguistics is the whole symbolism of mathematical representations. Certainly in algebra there's an awful lot as we talk about unknowns. These really are represented with symbols and a lot of processes are represented by symbols. And so symbolism in a sense is language – all language consists of symbolic representations and mathematics is just a variation on that theme.

> ***Need for students to explain their thinking (expressive language)***
> And then finally a part of math that sometimes unfortunately under-utilized but is really critical is the expressive language aspect of mathematics – which is to say that children need to keep explaining how and why they did what they did to solve a word problem or some other calculation in mathematics. One of the best ways to learn something is to have to teach it, so mathematics students in a way should be amateur mathematics teachers. They should constantly provide explanations for what they've done, and when a kid says I did it all in my head, that's unacceptable. He has to be able to put it into words; otherwise he might be passing along with a very tenuous grasp on what he's doing.

> ***Phonological Awareness***
> I want to mention one more language aspect that's been studied recently which is that it appears that phonological awareness through mastery of language sounds actually participates in the process of learning math facts. Sounds odd, but when a kid says "three times eight equals twenty-four," that's actually a collection of sounds that can really get entered into phonological memory. It's not the whole story, but it definitely facilitates math fact learning. And it's been found that some kids with phonological problems have a bit of trouble learning their math facts.

Higher Order Cognition

Well as I mentioned, the second neurodevelopmental construct that contributes to mathematical proficiency is Higher Order Cognition, our most sophisticated level of thinking. A number of components of higher order cognition are worthy of mention.

> ***Conceptualization***
> First and foremost, there's conceptualization – a really in-depth understanding of the operations in math, the various entities such as different geometrical forms, the ability to really know what they are and to have deep insight into them. Concepts like place value, like the concept of a decimal, a fraction or a percentage, a concept that embodies a process like factoring or reducing a fraction. The way we like to think of concepts is that concepts consists of critical features. So we might say "What are the critical features of a decimal?", "What are the critical features of a fraction?", or "What are the critical features of an integer?" And to begin, kids really ought to make a list of what are the features of these things, and what are some examples of these concepts.

We have a number of different kinds of concepts. For example, there are ***verbal concepts*** –

these are concepts that are beautifully characterized with language in a way you don't need anything else except words to describe the concept. And then we have nonverbal concepts. ***Nonverbal concepts*** are very much aided by mental imagery, by visualization – really being able to picture or form images of some of the critical features of the concept so that the concept of a parallelogram could be well embedded if you were actually picturing a parallelogram and perhaps providing some linguistic critical features, but at the same time you're adding to those some imagery, and we would call those nonverbal concepts.

One of the most critical parts of conceptualization in math is your ability to thread the concepts. Almost every time you learn a new concept, you need in your mind to draw a dotted line or several dotted lines to other concepts. So do you really understand how fractions are similar to decimals which are similar to percentages? Do you see the relationships between multiplication and addition? Can you see the interactions between subtraction and division? And it's these kinds of linkages that solidify mathematical learning. There are some children who try and get through math without any conceptual base. Sometimes it's called the extreme algorithmic approach, you know how to do everything but it's unclear that you know what you're doing. This is a dangerous way to learn math, one that ultimately can deteriorate over time, so we hope that kids are understanding the concepts, really understanding the critical features of the concepts that come up in math.

> ***Problem-Solving/Dilemma Resolution***

Another big part of math is your ability to engage in problem solving. Problem solving in a way is a stepwise approach to coming up with mathematical conclusions, or decisions or solutions. It requires kids to slow down, to think through something before they do it and more than anything else, it's almost the opposite of an impulsive approach to math. It's a more reflective approach where you stop and say "Okay, what's the problem? Where have I seen this kind of problem before? What's going to be my plan for solving it? How am I going to watch myself doing it so I'll know if I'm getting it right or not?" And that ability to progress in a very deliberate logical fashion is what we might call problem solving skill. We also sometimes call it dilemma resolution, because really every math problem represents a dilemma. "What are the ways I can do this, and what's the best way?" That's the dilemma of mathematics and it requires a stepwise approach, one that is really thoughtful. I think sometimes kids ought to be encouraged to elucidate their problem solving tactics before they attack a problem.

> ***Mental Representation***

Another interesting component of higher order cognition that's certainly plays a role in math is mental representation. When you learn something new in mathematics, how do you portray it in your mind? How are you going to set it up so it's maximally meaningful and stable in your mind? When your teacher is explaining how an exponent works, how are you going to set that up? And it turns out that the best mathematical and science students, when they learn something new represent it in several different ways. They may put it in their own words, they may think of visual image for it, they may think of it in a formula, they may think of an example of a problem that has that particular entity in it, or they may think of some experience in life that they've had with that particular idea or concept has played a role, or it could be that they will think of an application for it in the future, think "Oh, that's cool. Here's how I'm going to use it." So it turns out that being able to represent something vividly in your mind and it's also the case that different kinds of minds have different forms of representation that work best for them. In fact I think that we can help kids in math if we enable them to understand how they best represent information.

> ***Applied Reasoning***
Another important part of math is called applied reasoning. Applied reasoning is your ability to make good use of logic. There are many different kinds of applied reasoning; some of them get applied in playing a sport or in doing artistic work. Other kinds of applied reasoning are needed for solving moral dilemmas. And in mathematics, there are really two kinds of applied reasoning that are critical. One is ***proportional reasoning***. If this recipe uses one and a half cups of flour and it's for 4 people, how much flour will I need to make the same recipe for 7 people? That's proportional reasoning – it plays a role in everyday life, and much of mathematics. Then there's also just general ***numerical reasoning***. Can you use the number system, the sequences of numbers, as sort of the basis of thinking? Reasoning is sometimes *deductive*, where in a sense you start with the answer as in a geometric proof and work backwards to how you got there. And other mathematical reasoning is *inductive*, where you're giving some information and you have to put it together to come up with an answer. So in a sense, reasoning can be defined as your ability to do something that isn't immediately apparent, or ability to solve a complex problem that's going to take some concerted effort and organization to solve.

> ***Higher Sequential Thinking – Sequential Conceptualization and Analysis***
And then I think we can add to our list of higher cognitive contributors sort of Higher Sequential Thinking – to be able to make use of all the sequences in math, and have a deep understanding of how they work, what their underlying mechanisms are. So higher sequential thinking really gets the basis of multi-step math processes, and our ability to think through these, and see how they are justified and what they lead to. Higher sequential thinking is also embodied in a geometric proof, in a sense that is the most elegant representation of organized higher sequential thinking.

Spatial Ordering

And our third contributor to mathematical comprehension is Spatial Ordering.

> ***Geometric Reasoning***
Spatial Ordering really has several sub-components itself, the first of which is to engage in geometric reasoning – to be able to take the different parameters and dimensions of the spatial world and think through them as they relate to various mathematical processes, such as calculating area, perimeter, and applying different formulae in geometric thinking. Somehow the ability to see what's going on in space and in those geometric forms you're manipulating certainty enables one to succeed with the more geometric aspects of mathematics.

> ***Three-dimensionality***
Also the capacity to think in three dimensions represents a crucial part of math and the ability to represent things and re-represent things in different forms, most particularly to be able to take various mathematical entities and represent them on a graph. The graphic kind of representation of mathematical processes or individual problems is key to Spatial Ordering.

> ***Visualizing***
It's also the case that kids need to be able to visualize while they're solving a problem in mathematics. That really is key, that capacity to run a kind of internal video in your mind. You know, it's been shown that children who are really good at math differ from kids who are

struggling in math in what they're visualizing while they're solving a word problem. A child who is struggling in math solving a word problem about two children trying to share some apples might really being picturing the color of the kid's hair, or what kind of apples they were, or whether the apples were on the floor, or whether they were on a table. Kids who are adept at math are actually visualizing the shifting of apples from one pile to another. The big difference is that the kids who are good at math are running an internal video where as the kids who are not so good are running a snapshot, a still [image].

And incidentally, in all of mathematics, in every mathematical operation, the basic subject matter is change. It's called semiotics – the study of change. And so that ability to visualize changes that are occurring in a mathematical process is a key part of spatial ordering and contributes in a big way to mathematical understanding. Virtually every study that's been done on kids who are good at math demonstrate that one of their attributes is that they have strong spatial, nonverbal thinking abilities.

If we refer back to our double helix, which is our need for having both understanding and recall in Math, we've now talked about the components of understanding, namely Language, Higher Order Cognition, and Spatial Ordering. We are now going to move on and deal with the other part of the helix, namely recall and the different factors, or neurodevelopmental functions that are impacting upon recall in Mathematics.

There are two major domains that we want to consider, namely **Memory** and **Attention**. And of course Memory and Attention are constantly linked to each other – it's hard to remember things you didn't focus on, and you may stop focusing if you're having trouble remembering. So, it turns out that there are multiple moving parts within both Memory and Attention and as we deconstruct math we need to dissect those out.

Memory

Let's begin with Memory itself. There are several different aspects of Memory we can refer to, the first of which is the balance between ***factual recall*** and ***procedural recall***. It's been known for a long time that in the human brain, there are two distinct kinds of memory pockets. One is your ability to recall facts, sometimes called declarative memory. It's called declarative memory because you can declare those facts. The other is procedural recall, or non-declarative memory, because you can't really declare how to tie your shoelaces.

> ***Factual Recall/Declarative Memory***
So with that in mind let's begin with factual recall. Mathematics of course, particularly in the earliest grades, requires an enormous amount of factual memory, in particular, learning your math facts, and even more specifically, your multiplication facts. In fact, it may be that learning math facts is the purest test of Memory there is. In my opinion, any kid who has major problems learning math facts really has an underlying Memory problem. And very often, the kid who has trouble learning math facts is also having problems with writing and spelling, two other academic areas that drain Memory.

> ***Procedural Recall/Non-Declarative Memory***
Then there's procedural recall. In a way, procedural recall is a form of sequential memory – it's your memory for processes, being able to recall the steps in long division, being able to remember the steps required to calculate the circumference of a circle. So the different processes in math require procedural recall. One of the unique aspects of math during childhood is that it is one of the few subjects where you have to simultaneously excavate information from both factual and procedural memory.

> ***Progressive Automatization***
Another critical part of memory is called progressive automatization. What that suggests is that as kids progress through mid- to late-elementary school, more and more of their factual recall

has to be on automatic pilot. They can't stop and think about it, it has to really come forth easily, quickly, and with virtually no effort, in fact. When you're in sixth grade and you have to stop and think about how much is two times four, even if you get the answer right you could fail math. The reason to have things automatized is so that you can pile more things on top of them. As math gets more sophisticated, there have to be some automatic processes going on so that you can engage in the more sophisticated components with all your intellectual resources. You can't have the bottom layer interfering.

It's also the case that math requires a lot of what we call Active Working Memory – you have to keep several different things in your mind at once. When a kid has trouble with Active Working Memory, his parents might observe that in the middle of a math problem he forgets what he's doing; he can't remember what he's already done and what he's intended to do. Sometimes Active Working Memory is negatively affected by weak automatization because certain facts have not been mastered automatically. It makes it hard for a child to keep other things in mind, so he loses part of a math problem because he's working too hard on the math facts or on a process that should have become fully automatized.

> ***Pattern Recognition and Method Transfer***
Another memory contributor to Math is called pattern recognition and method transfer and this is really key. Pattern recognition is your ability to look at something and know that you've seen it before, to be able to look at a word problem and something lights up in your head that tells you you've seen that kind of word problem before. Pattern recognition has a partner called method transfer, which is to say that you recognize a pattern and you transfer over the method that has worked in the past when you've encountered that pattern. Pattern recognition and method transfer are really key. I do want to mention though that when patterns come back again in school, especially in math, they don't look exactly the same as they did last time. This is the first time in your life that you've ever seen a word problem about a rabbit and a mouse. It's the exact same word problem you saw last week, but then it was about a car and a truck. Can you peel away the superficial difference to get at the underlying pattern? Some children are so deceived by the superficial differences that they fail to see that underlying pattern, so they can't transfer over the method that works when one comes across that pattern. Pattern recognition and method transfer: key in mathematics, and also in quite a few other subjects. Teachers have to keep saying, "What's the pattern here and have you seen it before?"

Attention

Now I'd like to move to Memory's close partner, called Attention. There are a number of components of Attention. Since Attention is very much the administrator of the brain, it's going to play a big role in mathematical competency and sort of habits of mind that can help in Math.

> ***Saliency Determination***
I want to just mention several different components of Attention that play a role in math proficiency, one of which is called saliency determination. Very closely related to the ability to focus on details, it's the selectivity of attention. When you look at a problem in math, can you pick out the key features that you are going to really need to be able to focus on to solve the problem. A big part of mathematical problem solving is your ability to hone in on details. There are some kids who much prefer the big picture to the details, and they do poorly in math because even though they may understand the concepts, and are pretty good at comprehending, on the memory side of the helix they're having trouble because they don't

focus and retain the little details. This might be a kid who doesn't make the distinction between a plus sign and a minus sign, that's just a detail, and he's a big picture guy. It may be a student who doesn't distinguish between the number twenty-three and the number thirty-two, those are just details, and so a kid who has trouble assimilating details, who's kind of detail-intolerant, may struggle in math.

> ***Production Controls***

There's also a set of components of Attention that originate in the pre-frontal cortex that we call the production controls. These production controls really add quality to what a kid is doing, and they play a major role in mathematical proficiency and efficiency. I'll mention a few of those.

One of the production controls is your ability to look ahead and foresee an outcome before you do something. It's called ***previewing*** and it turns out that previewing plays a substantial role in mathematics, especially when it comes to estimation skills. In fact, of people who have had surgery on the pre-frontal cortex, one of the first things they lose is their estimation skills in mathematics – having a rough idea of how a problem will turn out.

The production controls also entail ***self-monitoring*** – knowing how you're doing while you're doing something, and knowing how you did right after you did something. Of course that ability to self-monitor and self-regulate is going to impact significantly on mathematics. Can you tell when you're going astray in the middle of a problem? Do you detect when you've made a careless error so you can go back and correct it again? Can you look back over something you've done and make it better? All of that is part of self-monitoring, and self-monitoring has a partner called self-regulation, namely, when things aren't going right, you can right them so that you're back on course.

Another part of the production controls is ***pacing***. One of the enemies of mathematical accuracy is impulsivity – kids who just go through a math problem frenetically. They're not slowing down and thinking as they're operating. Some may just want to get it over with, while others really may have a kind of attentional difficulty in which they have so much trouble modulating the pace at which they do things. And if you do math too rapidly, you could lose out, you could make a great many careless mistakes. So that pre-frontal cortex really becomes a strong ally of mathematical learning when all is going well.

So it is that we've looked at these different neurodevelopmental functions, embedded within our neurodevelopmental constructs that have a big impact on math. And these particular brain functions really are the determinants of whether a child will be able to recall what he learned in math, and therefore be able to apply it, and just as importantly, whether he's going to develop adequate or, at least, usable mathematical comprehension. Namely, will he know what he's doing? And so much of his future success in math will depend on both understanding and remembering.

Elementary math concepts as a platform for sophisticated mathematical thinking and application

Mathematics occurs in layers. It's kind of hierarchical, such that really children as they're learning math have to almost act as there jumping over hurdles at a track meet. The challenges of math kind of splay out over time in a fairly predictable and systematic way, and a child has to face each challenge, master it, and then be able to go on to the next. Let me try to articulate the seven major challenges or layers of mathematics that children encounter in elementary school.

> ***Number Sense***
The very first is called number sense, really having a grasp on how numbers work, knowing that when you count something, the last number you mentioned is the number of objects. Being able to deal with number lines, being able to see the relationships between numbers, and how the symbolism of numbers translates into specific objects. A deep sense of number then is really the first challenge and it often occurs during the earliest grades in elementary school.

> ***Number Combinations***
The second challenge is the mastery of mathematics facts and number combinations, as they are sometimes called. These are just the basic addition facts and multiplication facts that ultimately have to be mastered and become fully automatic. So to be able tame the giant of math fact learning is really our second challenge.

> ***Basic Operations***
And then there are basic operations – the most fundamental processes in mathematics that includes your ability to add, to subtract, to do simple multiplication problems, and simple division. It also has a pre-algebraic component, when kids are able to solve problems like "six plus what equals eight", and those kinds of pre-algebraic processes are included in the basic operations that kids have to try to conquer as there moving through elementary school.

> ***Geometric Sense***
Above basic operations we can include geometric sense – beginning to look at the world of space, its various parameters and dimensions, and shapes and their interactions and relationships that operate, in a sense, in a geometric world. The mastery of basic geometry is really a significant challenge, one that will have a payoff if it's met as kids take more advanced geometry courses, and also as they apply mathematics in various scientific subjects.

> ***Data Display/Analysis***
The next level is really data display and analysis – being able to represent numbers in a variety of different ways. Probably the most striking example is being able to graph different processes and display them in that way. Also to be able to analyze data, to engage in some basic statics, and thinking about probabilities would be included in this level of mathematical challenge.

> ***Word Problems***
The next level is word problems, and this is a real barrier for certain students, but it's an essential part of mathematical learning because it kind of bridges mathematics with the practical world, and it bridges the abstract with the concrete. It really enables students to see that what they're learning has highly specific applications. Word problem mastery is really in many respects one the many accomplishments of elementary mathematical learning.

> ***Fractions***
And the final sort of finishing step as kids progress through elementary mathematics is their ability to deal with fractions, ratios, and proportions; really understanding what a fraction is, how fractions operate, how they play a role in word problems, and in every day life, and how fractions relate to decimals and percentages. And that mastery of fractional thinking is really one of the final requirements if kids are going to be competent at mathematics.

An essential infrastructure for Math beyond elementary years

The things I've just mentioned – number sense, math fact mastery, basic operations, geometric sense, data display and analysis, word problems, and fractions – become the absolute infrastructure for math beyond the elementary years. They are an indispensable foundation if kids are going feel comfortable and competent at mathematics in secondary school. I believe that for each of these seven layers or challenges, kids in a way need to be certified. They need to absolutely demonstrate not only can that they apply something at that level, but that they really and truly understand it; that they can recall facts, processes, or concepts at each level, and that they can relate different components to each other – they can draw those kinds of horizontal threads between components. So in a way, one far out idea might be that instead of giving kids grades in mathematics, they would get certificates when they would demonstrate competency in each of these seven layers of Math.

Cumulative effects and risks of tenuous grasp at any level

Well moving on, we also have to recognize how cumulative all of this is. Earlier I mentioned cumulative memory and its impact on learning mathematics and its potential intimidating qualities. There have been some studies that have shown kids who are more than 6 months delayed in math in 6th grade will never catch up. So many difficulties with Math in high school can be traced back to these challenges. This is so powerful that you wonder instead of thinking of the different mathematical challenges as a sequence in learning, whether we really ought to keep circling back and be recursive, whether somehow in 5th grade or 6th grade we should go back and cover number sense again. Whether as kids are really advancing way beyond the elementary basic operations, we should go back and review them and talk about them to make sure that each of these levels are firmly consolidated, integrated and internalized in a kid's mind so that he won't be shaky or anxious when it comes to later life.

Applied use in secondary math, in the sciences and everyday life

It's also the case that as these different challenges are mastered, students need to see how they are going impact on further learning and how they are going to play a role in the appreciation of the sciences, and in everyday life at the mall, at the supermarket, and even at home to develop budgets and think about different family economic issues. So, I think it's important to address these different levels, to evaluate how kids are doing in them, to be very explicit in the way we teach mathematics at these levels, and to keep circling back and re-establishing the permanence or stability of each of the challenges that kids have to master in elementary school.

How do we manage the wrath of math? How do we deal with its intimidating features? How do we enable certain children who are struggling in math to find some success in that area? It's been found that during elementary school, kids have trouble deciding whether they have intellectual ability or not. It seems to vary almost from day to day. Sometime during middle school, during early adolescence, students decide whether they're smart or not, and when you interview these students, you find out that one of the major criteria they deployed to decide whether or not they had intellectual ability was how they were doing in mathematics. Math was seen as kind of the community I.Q. test on the part of a lot of kids. And so, there's a lot at stake in terms of how accomplished, how successful children are when they tackle mathematical learning, and a lot of anxiety, even depression can ensue when a kid feels totally inadequate at mathematics. It takes a big chunk out of a student's self-esteem. So, I think we have to make a concerted effort to make math less of a threat, and to help more and more kids feel more comfortable and competent when it comes to mathematical learning.

Demystification

I think when a kid is having trouble learning math, it's vitally important that we demystify that kid. Demystification is the process through which we talk to a child, through which we give kids the vocabulary they need to think about any issues they're having in school. So, for example, if we had a child who was doing poorly in math because of some Memory-based weaknesses, say problems in Active Working Memory and pattern recognition memory, we might sit down and explain how those processes work and get the child to the point where instead of saying, "Boy am I a dummy, I can't do anything in math," he can say, "My weakness in pattern recognition is taking is toll on my mathematical ability. I have to work on my pattern recognition so I can do better in math." Demystification is critical, and incidentally the demystification should also include strengths, it should be basically optimistic, upbeat, and encouraging, so kids don't sort of sign out of math, or rule out any possibility that they could ever succeed in that subject area.

Math affinities, intuition and motivation

I think also it's really important that kids are actually identified when they have real affinities for math, so that we can keep them stimulated and interested. I think we can also help kids learn how to use intuition in math. By the way, intuition is a mixed blessing; it's both good and bad. Some students are just highly intuitive and they can even learn calculus and other advanced math on intuition which seems to make some kind of internal sense to them. But intuition can get in the way of analytic thinking, as well. And we have to be a little nervous about that. Let me mention a problem that's been used in a lot of research studies that helps people distinguish between intuition and real analysis or problem solving skill in math. Listen to this: A baseball and a bat cost $1.10 altogether. The bat costs $1 more than the ball. How much does the ball cost? Most people who are highly intuitive would say the ball costs $0.10. The truth is, if you analyze it and don't operate on intuition, the ball costs $0.05. (By the way, I missed that, because I'm very intuitive.) Kids have to be able to distinguish between their intuitive thinking and analytic abilities.

Motivation also plays a big role. Kids are motivated towards something if they find the goal attractive, and who wouldn't find an "A" in math attractive? They're motivated towards something if they feel they can attain the goal. So you're not going to be motivated in math if you believe that there's no way you can succeed in math, and you're just going to lose motivation and that will of course ensure that you're not going to succeed in math. It's also they case that you're motivated towards something, if you think you can do well in it, without superhuman effort, so if it looks as if it's going to be too hard, it's going to drain you of all of your effort, you might motivationally sign out of mathematics. So once again we have to keep kids optimistic and more than anything else, they have to get away from the notion that you have to be smart to do math, so if you're having trouble in math maybe you're not smart, and instead, really begin to embrace the truism that effort has a payoff, that if you stick with it, if you work with it, if you really stay focused on math you can improve substantially.

Alignment of student's profile to expectations

I also think as we manage math issues, we have to try to look at a kid's profile, his neurodevelopmental profile and his sub-skill profile in math, and align this with expectations. If we're going too fast, if we're way out ahead of a kid, he can become overwhelmed, anxious, and even phobic about mathematics.

Leveraging Strengths

So we really have to keep matching the profile to expectations, and in particular, leverage a child's strengths. Let me tell you what I mean by that. Recently I saw a little girl who was having a lot of trouble in math, but she was one of the best students in her English class. She was extremely verbal, just a marvelously fertile participant in class discussions. What we did with her was every time she learned something new in math, we asked her to put it in her own words, to pretend she was teaching it to her little sister. In fact, several times she even made recordings of her explanation, of her recoding of what the teacher said, and the more she verbalized math, the more successful she was at it. Then we've had students who are just the opposite, who are not very verbal, but are very good at spatial thinking. And these are kids who do their best if while they're learning math, keep a kind of log with demonstration models of different problems, because they learn best visually. If anything they get confused by verbal explanations or verbalizations of math. They need to see it rather than hear it, they need to dissect a correctly solved math problem and figure out what's going on. So those strengths in the spatial domain or in the language area can really be used to leverage mathematics.

We also have to tease out some of the key processes in math, and sometimes end a problem in the key step in the problem. Let me give you an example. From time to time, it might be good to give kids word problems and tell them they don't have to solve them; they just have to identify the pattern in each word problem and put a circle around it. "Can you find a word problem whose pattern is multiplication and circle the words that gave it away?" In this way students can improve their pattern recognition, and by stopping it there without actually solving the problem, we can accentuate a particular step that's crucial like pattern recognition. I think kids also, particularly those having trouble in math, ought to be graphically representing the concepts, kind of keeping an atlas of the different mathematical concepts. This can be particularly useful for kids who are struggling if they use some kind of software that can graphically represent concepts, their critical features, and their dotted lines to other concepts. This can help solidify concepts in students' minds.

Career planning

And finally, kids need to recognize the potential role that mathematics might play in the various careers they are considering. I think a lot of them are fairly naïve about this. I remember last year I met a boy and asked him what he wants to do, and he told me he wants to be a marine biologist. This was a kid who loved going and seeing the coral reefs in various places when he went on holiday with his parents. He said he was really interested in that, but on the other hand he was failing math. He thought that as a marine biologist he would spend every day at work snorkeling and somehow didn't recognize how much mathematics there is in marine biology. So I think sometimes kids are a bit naïve about the fact that they need some mathematics if they're going to go into business or if they're going to go into medical school. In fact, if they feel really intolerant or negative about math, need to discover career pathways that minimize mathematics, or perhaps even work styles where you collaborate with someone who's good at math. That person will do the statistics; you'll generate the hypotheses and methods. So math is a good place for collaboration as well.

Nevertheless I think we can say that it can be awfully empowering and certainly constructive for kids to taste some success in mathematics. I firmly believe that if we understand the child's strengths and weaknesses, and if we get in on the ground floor and don't let a student become too far behind or feel as if she or he is actually drowning in mathematical knowledge, I think we can turn every kid around and make every child more comfortable in the world of quantification and change that mathematics so vividly represents.